COMPLETE FUSION AND ITS LIMITATION

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Résumé. Un essai de définition de la fusion complète est fait dans l'introduction. Parmi toutes les collisions profondes qui passent par un intermédiaire composite, on fait la distinction entre les collisions très inélastiques pour lesquelles la fusion se produit déjà, les quasi fissions où l'amortissement de l'énergie est total et la fusion complète pour laquelle les deux noyaux sont agglomérés pendant un temps assez long pour que l'intermédiaire décroisse en produits finals sans souvenir de la composition du projectile et de la cible. Cette fusion complète peut, dans certains cas, être distinguée de la formation du noyau composé si la desexcitation a lieu avant équilibre total dans tous les degrés de liberté. Une évolution continue a lieu depuis les collisions légèrement inélastiques jusqu'à la formation du noyau composé.

Dans le paragraphe 2 on montre comment la mesure des sections efficaces pour la fusion complète entre noyaux légers à faible énergie peut permettre de définir une barrière d'interaction pour la fusion. Une tentative d'explication est donnée pour les oscillations des fonctions d'excitation pour $^{12}C+^{16}O$. Puis, à plus haute énergie, une limitation plus importante intervient due au moment angulaire orbital dans la voie d'entrée et le concept très utile de distance critique est expliqué (Galin et al.,et Bass). La détermination de $\ell_{cr}\hbar$ à partir des fonctions d'excitation et l'application du code Alice sont discutées. On montre pourquoi la descente vers les hautes énergies des fonctions d'excitation (IL,xn) dépend de façon très sensible d'une certaine limite au moment angulaire. Mais cette limite n'est peut-être pas toujours la valeur ℓ_{cr} pour la fusion complète car les hautes valeurs de J conduisent à l'émission de particules α.

Ensuite, pour les systèmes lourds, la distinction est faite entre fission après fusion complète, fission de prééquilibre et quasi-fission. Le modèle de la barrière de fission de goutte liquide tournante est discuté. Enfin, l'hypothèse d'une limite aussi du côté des faibles ℓ est exposée, à partir des résultats expérimentaux sur le déplacement des fonctions d'excitation. Les autres tentatives d'explication ont échoué.
Pour conclure, on montre combien la fusion complète dépend de la dissipation d'énergie et en quelque sorte d'un équilibre entre les forces coulombiennes (conservatives) et les forces dissipatives.

Abstract. First, a definition of complete fusion is given. Amongst "hard" collisions which pass through a composite system, a distinction is made between deep inelastic collisions where some fusion process occurs, quasi-fission where a complete damping is attained, complete fusion and compound nucleus formation. Complete fusion corresponds to interactions where both partners are joined together a time much longer than the collision time and make an intermediate which decays into the final products without particular remembrance of the composition of projectile and target. It might differ from compound nucleus formation as far as the full equilibrium before decay is not required. It is shown how there is a continuous evolution between smooth inelastic collisions and compound nucleus formation.

An analysis is made in section 2 of the deduction of the interaction barrier for fusion (incomplete and complete) from cross section measurements in the cases of light and medium systems at low energies for which fission is a negligible process. An attempt is made to explain oscillations in the excitation functions for σ_{CF} in ($^{12}C + ^{16}O$).

In section 3 the limitation to complete fusion due to high orbital angular momenta and the very useful concept of critical distance are explained (Galin et al.,and Bass). The basic concept of the Alice code is discussed as well as the determination of $\ell_{cr}\hbar$ from excitation functions. It is shown that the slope of decreasing branch of

(HI,xn) excitation functions on the high energy side depends very strongly on some type of angular momentum limit. But it might not be the critical value for complete fusion, because high J population decays mainly by α particle emission.

In section 4, the distinction between fission after complete fusion, preequilibrium fission and quasi-fission is made for heavy nuclei. The concept of rotating liquid drop fission barrier is discussed. The hypothesis of a limitation to complete fusion from low ℓ-vawes is exposed, as well as the experimental results on excitation functions that has been obtained. It is shown that all other explanations fail.

As a conclusion, it is shown how complete fusion depends strongly on the energy dissipation and on the balance between conservative Coulomb forces and dissipative forces.

- COMPLETE FUSION AND ITS LIMITATION -

1 - Introduction. "Hard" collisions, fusion, composite system, complete fusion and compound nucleus formation.

A rather popular joke in a number of recent meetings on heavy ion physics was to talk about "confusion" when the subject of complete fusion between two complex nuclei was discussed, and that should have perhaps restrain me to accept to treat again this question.

As originally used, the expression "compound nucleus" that some authors consider as equivalent to "complete fusion", was applied to a nucleus excited by the absorption of a single nucleus and excited to a single level, and existing long enough for the mode of decay to be independent of the mode of formation. For a nucleus excited to an energy region where many and many levels overlap, the time should be long enough that a thermodynamical equilibrium should be attained. The formation process and the decay process, being separated by the intermediate existence of the compound system, should be independent except for the restrictions imposed by the conservation of energy, total angular momentum and parity. The compound nucleus should exhibit equipartition in the occupation of all accessible degrees of freedom. For high excitation energies, the quantitative treatment was made through the well known Weisskopf's theory [1] and the angular momentum effects have been included in the level density estimation by Hauser-Feshbach [2], Ericson [3], Thomas [4], Grover [5] and many others.

In principle, absorption of a complex projectile by a complex target means that, after a while, nucleons from the projectile lose their previous collective and individual characteristics and take on new characteristics in a single nuclear potential. All the kinetic energy allowed by momentum conservation is distributed amongst all accessible degrees of freedom. Therefore, it should be rather easy to observe such a system resulting from complete absorption. However the problem of time scale for highly excited compound nuclei becomes predominant. It is difficult to estimate very accurately the life time of excited compound nuclei. For low and medium excitation energies, and low angular momenta, experimental measurements have been performed with the help of a very beautiful technique using the blocking effect in crystals [6]. A fairly good agreement between the results and the previsions of the statistical theory was found, in the life time range between 10^{-16} and 10^{-18} sec. For higher energies, lifetime predictions can be made according Ericson[3]'s formulation in the case of neutron evaporation.

The life-time is the inverse of the neutron emission probability, and therefore is proportional to the ratio of the level density in the compound nucleus to the level density in the residual nucleus. Table I indicates some estimates which I have made under a classical model of spin dependent level density formulation.

Table I

Estimated life time for compound nuclei
A ≃ 180, a = 22.5 MeV^{-1}

E^*(MeV)	J(ħ units)	t(sec)
100	60	10^{-20}
100	10	2.10^{-21}
50	60	10^{-19}
50	10	3.10^{-20}
30	60	10^{-16}
30	10	10^{-18}

Neglecting the effect of angular momenta, Blann [7] has estimated that at excitations greater than 100 MeV, for nuclei of A = 100, the life time is of the order of the relaxation time assuming a binary nucleon-nucleon energy transfer mechanism (5×10^{-23} sec)

The time required for two A = 50 nuclei to pass one nuclear diameter was estimated around 10^{-22}sec. During that short time, the two complex nuclei are fusing. This implies that particle emission may take place while this short fusion time, but before equilibration. Such precompound decay has been estimated by Blann for two systems : $^{12}C + ^{141}Pr$ and $^{40}Ar + ^{109}Ag$ at various excitation energies. Specially for the lightest projectile and the highest excitation energy, a non negligible fraction of the particles are coming out during the short fusion period, but before equilibrium ; although the number of nucleon-nucleon collisions taking place is rather large. When more than one neutron or proton per nucleus is emitted in a fast process, it means that there is never a complete compound nucleus produced. Should we call however such a phenomenon Complete Fusion ? My answer will be yes, if one restricts the definition as follows : Complete Fusion corresponds to interactions where both partners are joined together for a time longer than the collision time and make an intermediate which decays into the final products without particular remembrance of the composition of the projectile and target. Therefore, when one measures the cross section for nuclei collected in the forward direction, with masses a little smaller than the sum ($A_1 + A_2$) of projectile + target, and with full momentum transfer, it corresponds to σ_{CF}, when fission can be neglected. If some particles are emitted very shortly in a precompound stage, then σ_{CF} is larger than σ_{CN}, and one might make a distinction between the two terms.

Another more serious difficulty arises when the fission process competes in the de-excitation of the compound nucleus. With light projectiles such a competition becomes important only for very heavy compound nuclei, because the orbital angular momentum cannot be very great. But it is well known that large orbital angular momenta are obtained with heavy projectiles and then a great enhancement of the fission process occurs. Consequently, compound nucleus formation is never observed directly as an amalgamation product, but two fragments are emitted shortly. However, there is in principle a way in order to distinguish between two fission fragments issued from a compound nucleus and two emitted partners resulting from an inelastic scattering. It is based on the momentum conservation law, and it has been proposed for the first time by Sikkeland et al.[8]. In the bombardment of uranium by oxygen ions, coincidences were counted on two detectors located at correlated angles in the laboratory corresponding to two fission fragments emitted at 180° in the system of the recoiling compound nucleus. The width of the correlation is related to the number of neutrons emitted by the fragment, but a clear separation could be made from fission events issued from an alpha particle stripping reaction, since less momentum was transferred to the uranium + alpha recoiling system.

However, there is a controversial discussion on this question, whether or not the observation of a full momentum transfer is a valuable criterion for transient compound nucleus formation. It has been argued [9] that a "composite system" may disintegrate shortly into fission fragments without passing the stage of a definite nucleus and then a full transfer of momentum would have occurred without a full thermodynamical equilibrium. How is it possible to distinguish between such a preequilibrium fission and a "true" fission ?

One of the proposed evidence for the existence of an intermediate stage in which a random equilibrium has occurred is the angular distribution symmetric around 90° and following in the centre of mass system a law $1/\sin\theta$. It means that the intermediate system has lived during at least one rotational period, so that the probability for disintegration into two fragments per unit angle should be a constant with regard to θ, the angle between the beam direction and the fission axis. Halpern and Strutinsky [10] and Halpern [11] have shown that, when the angular momentum of the compound nucleus J is much larger than its projection on the axis of symmetry of the fissioning shape, the distribution $W(\theta)$ approaches $1/\sin\theta$. Such a distribution should be observed for all, heavy and light, fission fragments. Moreover, there is no special reason to believe that the time for a complete rotation is connected to the time necessary for a full equilibration. Probably, equilibration for a number of degrees of freedom occurs before a complete rotation.

My opinion is that, when mass and energy distribution of fission fragments behave just like those resulting from well established fission reactions, i.e. symmetric mass distribution, kinetic energy corresponding to Coulomb repulsion at the scission point, etc.., and when the $1/\sin\theta$ angular distribution is observed over the entire mass and Z distribution, one should consider the phenomenon as passing through at least the stage of Complete Fusion, if not fully equilibrated compound nucleus.

However we feel here the difficulty for making very strict classes of phenomena. In a heavy ion, "hard" collisions, after the target and projectile touch, the nuclei remain in contact and strong dissipative forces operate. The system has amalgamated into a composite nucleus which might be compared to the saddle shape along the fission path, although the fusion path is generally distinct from the fission path. Then the composite system might either separate into two fragments with substantial mass transfer and energy relaxation. This is the well known deep inelastic collision or quasi-fission process that shall be described by Galin [12] and Moretto [13]. Or the system moves towards a less deformed shape and statistical equilibrium occurs. The well ordered energy has been dissipated totally into many degrees of freedom. But if the nucleus is heavy enough, the decay consists of fission fragments. Different processes are sketched in figure 1 which is presented as a general introduction for the three reports on complete fusion and DIC.

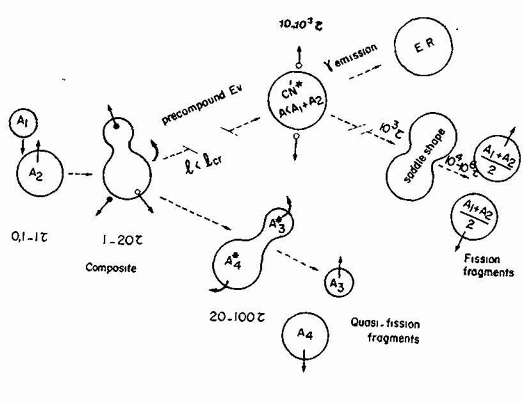

Fig. 1 - Schematic description of different processes occurring for "hard" collisions passing through a composite system. The time scale is $\tau = 10^{-22}$sec.

Again, there is a problem of time-scale, which can be reflected in the angular distribution since all the system rotates. When initial conditions are very asymmetric (light projectile, very heavy target), quasi-fission fragments exhibit a mass distribution around the projectile and target mass, although *fission* after complete fusion shows a well known symmetric mass distribution. But when one starts with a system close to symmetry, the distinction becomes impossible. And indeed its signification vanishes. The same intimate contact of amalgamation, the same basic processes of large local energy dissipation occur.

I have borrowed to L. Moretto and J. Sventek [14] a very nice illustration (Fig.2) of that discussion.

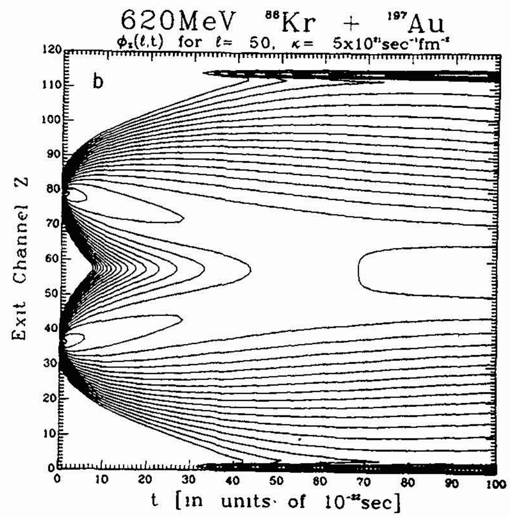

Fig. 2 - Calculations according Moretto and Sventek's diffusion model [14]. After a delay of 50.10^{-22}sec symmetric division into two equal mass fragments occur. Is it "complete fusion" followed by fission or quasi-fission after long relaxation time ?

Applying their diffusion model they have calculated for the initial system ^{86}Kr + ^{197}Au, the probability distributions along the mass-asymmetry coordinate as a function of time. The continuous trend between two quasi-fission fragments around Z = 79 and Z = 36, and two equal Z fragments around Z=58 is shown. In that respect, should we call complete fusion what occurs after 60.10^{-22}sec, since it happens to be roughly equal to one rotational period for ℓ = 100 and, on the figure, do longer times correspond to symmetric mass distribution ?

2. Large complete fusion cross sections nearly equal to the total reaction cross-section. Interaction barrier for complete fusion.

2. 1 - Cross sections versus Energy

At low incident energies, and at least for the following projectiles : ^{12}C, ^{14}N, ^{16}O, ^{20}Ne, ^{32}S, ^{40}Ar, and ^{35}Cl, and for targets lighter than ^{90}Zr (^{27}Al, ^{48}Ti, ^{52}Cr, ^{58}Ni, ^{60}Ni, ^{62}Ni, ^{74}Ge, ^{77}Se, and ^{90}Zr) the cross-section for complete fusion is approaching the total reaction cross - section.

The most recent measurements were made using a Δ E-E telescope located in the forward direction. This technique is able to assign masses of evaporation residues and therefore to separate the complete Fusion products from other reaction products, as far as the fission process is negligible. This is probably correct in the region of interest since recently Bisplinghoff et al. [15] have measured fission cross sections lower than 150^{mb} for ^{35}Cl induced reactions at energies up to 130 MEV (center of mass). Most of the excitation functions for σ_{CF} are similar to the typical one presented on figure 3 due to Scobel et al.[16]

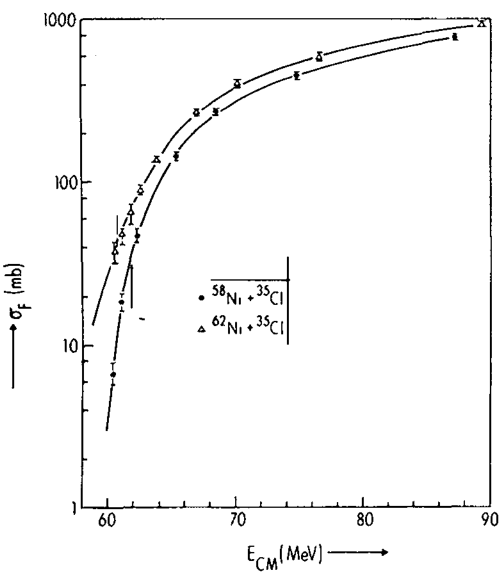

Fig. 3 - Excitation functions for complete fusion (evaporation residue) cross sections for ^{35}Cl induced reactions on two nickel isotopes (After Scobel et al [16])

A classical plotting of σ_{CF} as a function of $1/\bar{E}$ results in a straight line, and is justified by the following consideration.

The problem of the interaction barrier for complete fusion has been discussed and clarified by Wong [17] and the meaning of the data extracted from experimental results is given in figure 4.

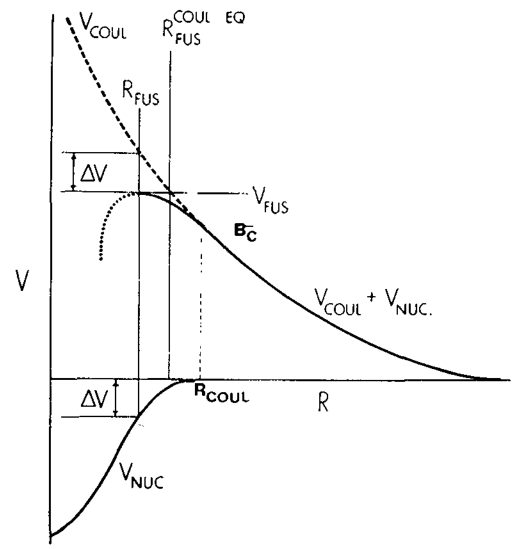

Fig. 4 - Potential energy versus distance, R, between two complex nuclei. Complete fusion occurs only when V_{Fus} is overcome. ΔV is the nuclear potential at the distance R_{Fus} where dV/dR = 0. $R_{Fus}^{Coul.Eq}$ is the equivalent radius obtained applying a pure Coulomb repulsion.

The nuclear, Coulomb and total potentials for $\ell = 0$ are drawn as a function of the internuclear separation distance. Going from the right to the left, at the distance R_{coul}, weak nuclear interactions should begin to occur, and the Coulomb potential is not yet decreased by the nuclear potential. Applying the partial wave summation, one may express the total reaction cross section

$$\sigma_R = \pi \lambda\!\!\!^{-2} \sum_{l=0}^{l max} (2l+1)\, T_l \qquad (1)$$

with the penetration factor T_ℓ approximated by the transmission of an inverted parabola (Hill and Wheeler) [18]

$$T_l = \left(1+\exp\left(2\,\frac{B_c-\bar{E}}{\hbar\omega_0}\right)\right)^{-1} \qquad (2)$$

where $h\omega_o$ is the curvature of the barrier for $\ell = 0$ and Bc is the pure Coulomb barrier.

Then

$$\sigma_R = \frac{R^2_{coul}\hbar\omega_0}{2\,\bar{E}} \,\mathrm{Ln}\left(1+\exp\frac{\bar{E}-B_c}{\hbar\omega_0}\right) \qquad (3)$$

is reduced to

$$\sigma_R = \pi R^2_{coul}\left(1-\frac{B_c}{\bar{E}}\right) \qquad (4)$$

when $\bar{E} - B_c$ is larger than $h\omega_0$.

The fusion barrier corresponds to a stronger interaction and to a smaller distance R_{Fus}. Let us define this barrier V_{Fus} at the top of the potential curve $(dV/dR = 0)$, at the distance R_{Fus} where complete fusion can eventually take place.

Figure 4 illustrates that there are certainly nuclear interactions between R_{coul} and R_{Fus} in the range where the nuclear potential V_N is not equal to zero. They do not lead to fusion phenomena, since the attractive forces are not large enough. However, it is not strictly correct to say that $\sigma_{CF} = \sigma_R$. The reaction cross section σ_R, as well as R_{coul} may be obtained by elastic scattering. Coming back to σ_{CF}, it may be expressed by the counterpart of equation (4) :

$$\sigma_{CF} = \pi R^2_{Fus}\left(1-\frac{V_{Fus}}{\bar{E}}\right) \qquad (5)$$

if all fusion processes end up into complete fusion

The nuclear potential at the distance R_{Fus} is written :

$$V_N(R_{Fus}) = V_{Fus} - \frac{Z_1Z_2e^2}{R_{Fus}} \qquad (6)$$

The intersect of the curve $\sigma_{CF} = f(1/\bar{E})$ with the ordinate should give πR^2_{Fus} and the distance R_{Fus}, as well as the intersect with the abscissa gives V_{Fus}.

The most complete results have been obtained by Scobel et al.[16] with ^{35}Cl ions and are plotted on figure 5. The extracted distances for complete fusion are slightly smaller indeed that the distances obtained from elastic scattering measurements according to the quarter point $(\Theta\ 1/_4)$ determination :

$$R_{sc} = \eta\bar{\lambda}\left(1+\frac{1}{\sin(1/2\theta 1/4)}\right) \qquad (7)$$

where η is the Sommerfeld parameter $\frac{Z_1Z_2e^2}{\hbar v}$

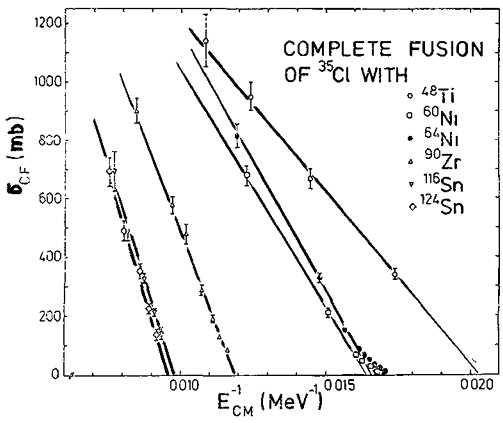

Fig. 5 - Complete fusion cross sections versus 1/E for ^{35}Cl induced reactions. (After Scobel et al [16]).

As an example in the reaction $^{58}Ni + {}^{35}Cl$, R_{Sc} = 11.03 fm, while R_{Fus} = 9,0 fm. If one expresses the distances R as a function of a classical parameter $r = \frac{R}{(A_1^{1/3}+A_2^{1/3})}$, then r_{sc} varies slightly between 1.58 for $^{35}Cl + {}^{27}Al$ and 1.50 for $^{35}Cl + {}^{124}Sn$, while r_{Fus} changes much more drastically from 1.34 for $^{35}Cl + {}^{56}Fe$ down to 1.20 for $^{35}Cl + {}^{116}Sn$.

The missing part : $(\sigma_R - \sigma_{CF})$ is of the order of 25 % of σ_R, a conclusion which contradicts the first asumption made by Gutbrod et al. [19] that the measurement of σ_{CF} could be taken as a determination of σ_R. The difference is assumed to go into inelastic and direct transfer reactions.

An interesting point is the threshold energy for both complete fusion and total reaction cross section. In other words, what are the first channels open when the Coulomb barrier is overcome ? There are not many precise results comparing the thresholds of quasi-elastic transfer reactions and complete fusion. First of all, when V_{Fus} and B_{coul} are very close because the nuclear potential is shallow at R_{Fus}, then

$$\sigma_{CF}/\sigma_R = \left(\frac{R_{Fus}}{R_{Coul}}\right)^2\left(\frac{\bar{E}-V_{Fus}}{\bar{E}-B_c}\right) \neq \left(\frac{R_{Fus}}{R_{coul}}\right)^2 \qquad (8)$$

and both thresholds are at the same value.
But for heavier systems where $(V_F - B_C = \Delta V)$ is significantly large, one should find a difference between the two thresholds, although the ratio σ_{CF}/σ_R should become smaller than $(R_{Fus}\ /\ R_{Coul})^2$ at higher energies. This has been observed indeed for the systems $^{12}C + {}^{152}Sm$, $^{16}O + {}^{150}Nd$ and

^{18}O + ^{148}Nd studied by Ishihara et al.[20]. Also, a gap of around 10 MEV has been observed [21] in the interaction ^{14}N + ^{209}Bi, between 1 proton transfer leading to ^{210}Po and complete fusion leading to an evaporation residue issued from $^{219}_{90}Th$.

2.2. Expression of the barrier for complete fusion.

Coming back now to relation [5], the extraction of experimental values of V_{Fus} has been compared with calculated potentials

$$V(R_{Fus}) = \frac{Z_1 Z_2 e^2}{R_{Fus}} + V_{Nuc}(R_{Fus}) \qquad (9a)$$

For example, the energy density formalism has been applied by Ngô et al. [22] to calculate the nuclear potential $V_N(R)$ in the sudden approximation making use of nuclear density distributions taken from Hartree-Fock Calculations. It contains no additional parameters. V_{Fus} is obtained by applying

$$\left(\frac{dV_N}{dR}\right)_{R=R_{Fus}} = -\left(\frac{dV_{coul}}{dR}\right)_{R=R_{Fus}} \qquad (9b)$$

From R_{Fus}, the parameter $r_{Fus} = R_{Fus}/(A_1^{1/3}+A_2^{1/3})$ has been deduced. It decreases monotonically when Z_1Z_2 increases, since the nuclear contribution has to be larger in order to compensate the Coulomb repulsion. In figure 6, are shown Ngo's calculated values [22] for r_{fus} as a function of $Z_1Z_2/(A_1+A_2)$ and also experimental points deduced from Scobel et al. recent data [16].

Although the agreement with experimental data was fairly good in the case of Ar, S, Cu and Kr projectiles, there are discrepencies with Cl induced fusion cross sections and also the variation from one isotope to the other is not reproduced.

The model of Krappe and Nix [23] is based on the liquide drop model with an improvement to account for the finite range of nuclear interaction. There are three parameters, the range a, of the nuclear potential, adjusted to 1.4 fm, the equivalent sharp radius parameter, adjusted to 1.16 fm and a distance d between equivalent sharp surfaces, which describes the overlap fo the two nuclear matter densities and therefore d decreases when atomic number increases.

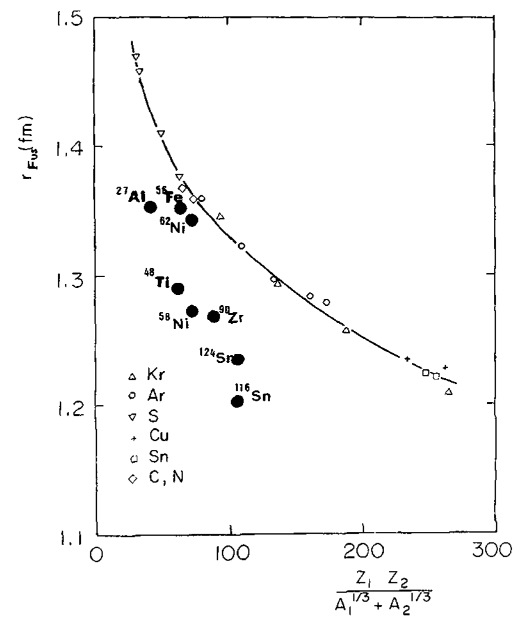

Fig. 6 - Plot of r_{Fus} (calculated by Ngô et al [22], versus $(Z_1Z_2/(A_1^{1/3}+A_2^{1/3}))$. The full curve represents the average behavior. Dark points are deduced from Scobel et al.'s experimental cross sections [16].

The fusion barrier is written

$$V_{Fus} = \frac{Z_1 Z_2 e^2}{r_0(A_1^{1/3}+A_2^{1/3})+a+d} \qquad (10)$$

In order to fit the experimental values of Scobel et al, d was adjusted between 1.6 fm for ^{35}Cl + ^{27}Al and 0.75 fm for ^{35}Cl + ^{124}Sn. This indicates that the nuclear density overlap at the fusion point slowly increases with increasing mass number. It is about 0.15 units of the saturation density for ^{27}Al and 0.27 units for ^{116}Sn.

2.3. Unexpected excitation functions. Oscillations in $\sigma_{CF} = f(E)$

Before to finish with this section, I should like to come back on the monotonic raise of excitation functions shown in figure 4 which I have quoted as a typical example. An another beautiful example is presented at this Conference by Eyal [24], for ^{12}C + ^{18}O, where C.F. cross section has been measured with an interesting technique based on the evaporated neutron counting (see also Eyal et al. [25]).

Nevertheless, there are a number of cases [26,27], namely $^{16}O + ^{12}C$, and $^{12}C + ^{12}C$, where the excitation functions for complete fusion exhibit oscillations shown in figure 7, while ($^{18}O + ^{12}C$), ($^{19}F + ^{12}C$) and ($^{14}N + ^{12}C$) have structureless curves. Therefore the oscillations cannot be attributed to some enhancements in σ_R for each successive resonating partial wave. The origin of this unexpected feature is not well explained. In the case of $^{12}C + ^{12}C$, one of the main de-excitation channel in the decay of the Compound Nucleus ^{24}Mg ends into ^{16}O. The excitation function for A = 16 follows the same oscillations as the total CF cross section [26]

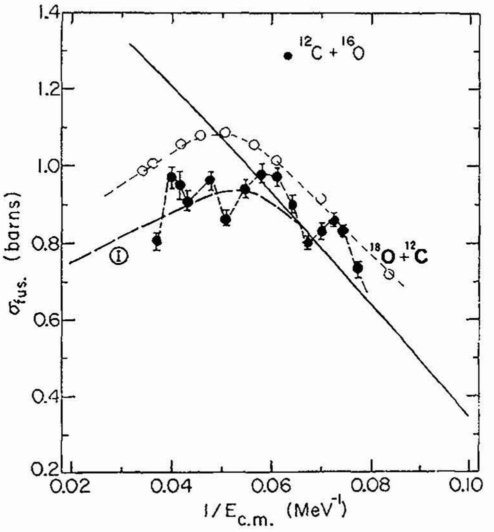

Fig. 7 - a) Excitation function for complete fusion in the systems ($^{18}O+^{12}C$) (After ref. 27). At low energies, σ_{CF} versus $1/\bar{E}$ follows a straight as predicted by equation(5). At higher energies the critical distance for complete fusion rules the cross section calculation according equation (13), with V_{cr} = -14 MeV and r_{cr} = 1.01 fm.

b) Excitation function in the system ($^{16}O+^{12}C$) where oscillations are shown (ref 27). The curve I was calculated according Glas and Mosel's model [37].

A possible explanation might be that Yrast states are reached irregularly when the excitation energy is increased so that the properties of these Yrast states may induce fluctuation - like structures, appearing mainly in nuclei like ^{24}Mg and ^{28}Si, as sketched in figure 8, and not in ^{30}Si or ^{26}Al. It is understandable that when the excitation energy is high enough to reach a bunch of Yrast states, there is an enhancement of the cross section. However, a further increase of E diminishes σ_{CF} because, although there are more partial waves involved in σ_R, the highest ℓ values which should participate to the cross section as $(2\ell+1)$, do not contribute to σ_{CF}, since there are yet no new bunch of states available in the compound system. This is a very interesting open question and I am sure it will be discussed at this Conference.

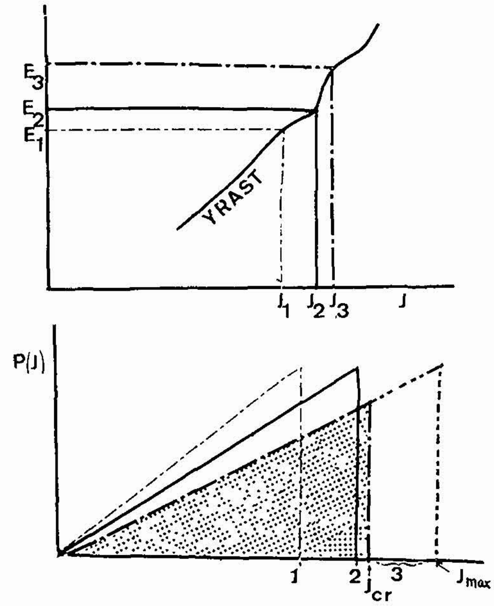

Fig. 8 - Schematic illustration of the effect of irregularities in the Yrast line on the variation of the complete fusion cross section versus excitation energy E^* (See text).

3. Limitations for Complete Fusion in light and medium nuclei. The concept of "critical distance"

3.1 - Results on Evaporation Residue Cross - Sections -

The preceding results, where σ_{CF} approaches σ_R, have been obtained only in certain particular conditions and do not represent at all the general feature of heavy ion reactions. The experimental observation that the evaporation residue cross section σ_{ER} as well as $\sigma_{CF} = \sigma_{ER} + \sigma_{fiss}$, was much smaller, has suggested, on the basis of partial wave summation, that there is a maximum orbital angular momentum in the entrance channel beyond which compound nucleus formation is inhibited [28]. A minimum mass overlap, corresponding to a minimum impact parameter should be reached before complete fusion occurs. The subject has been discussed at many occasions and I shall refer to the numerous reviews on the subject [29-30-31]. Also, a compilation of available σ_{ER} values is given in a recent report [32].

For most of the data obtained with ^{12}C, ^{14}N, ^{16}O, ^{20}Ne, ^{32}S and ^{35}Cl projectiles on light and medium mass targets, the fission cross section was small or negligeable, so that σ_{ER} is equivalent to σ_{CF}.

In this report, we shall focus mainly on more recent results and show that, except for perhaps one or two special cases, the nature of the limitation can be understood in term of the critical distance concept [33] , as described either by the Bass' model [34] or by the energy density nuclear potential calculations made by Ngô et al [22-35].

3.2 - Critical distance for complete fusion -

Let us only remind that the complete fusion cross section is related to the critical angular momentum by the relation

$$\sigma_{CF}=\pi \lambda\!\!\!^{-2}(l_{cr}+1)^2 \qquad (11)$$

expressing the summation of partial waves up to a limiting value $\ell_{cr}\hbar$. The first statement we make is that, for a given system, there is a linear dependence between ℓ^2_{cr} and the kinetic energy in the entrance channel, as shown in figure 9.

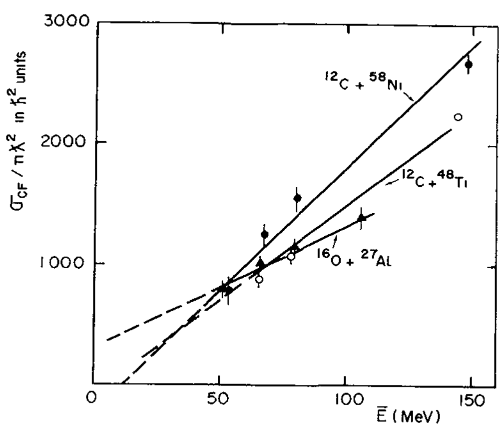

Fig. 9 - Plot of $\sigma_{CF}/\pi\lambda\!\!\!^{-2}$ versus energy for reactions $^{12}C+^{58}Ni$ (●), $^{12}C+^{48}Ti$ (○) and $^{16}O+^{27}Al$ (▲). The straight lines correspond to expression (11) in the text. (Ref.[32] .

In 1974, at the Heidelberg Conference [36], I have shown that one can associate a distance at the contact point for fusion, R_{cr}, to the corresponding impact parameter b_{cr} and one deduces, exactly as for the total reaction cross section,

$$b^2_{cr}\mu\bar{E} = \frac{1}{2}\, l^2_{cr}\hbar^2 = R^2_{cr}\mu(\bar{E}-V_{cr}) \qquad (12)$$

where μ is the reduced mass, and V_{cr} is the potential at the distance R_{cr}. Hence, the cross section for complete fusion is equal to

$$\sigma_{CF}=\pi b^2_{cr}=\pi R^2_{cr}(1-\frac{V_{cr}}{\bar{E}}) \qquad (13)$$

Such a formulation has been proposed at the same time by Glas and Mosel [37], who have shown that it corresponds to the asymptotic form at high energies a more general expression. On the low energy side, one obtains expression [4] $\sigma_R = \pi R^2_{coul}(1 - \frac{Bc}{\bar{E}})$.

With the help of expression [13], one can deduce from a plot of experimental values of σ_{CF} versus $1/\bar{E}$, the critical radius R_{cr} and the critical potential V_{cr}. These values may be compared with theoretical calculations for the potential at the distance where complete fusion occurs.

Let us now describe very briefly basic ideas for the theoretical estimations. The problem is to relate the repulsive effect of the angular momentum as expressed in the centrifugal potential to the distance between the two centers at which it operates. The possibility for a large number of intrinsic excitations ending up into a compound nucleus formation depends on two conflicting tendencies : attraction by nuclear forces which are more and more efficient when the distance diminishes ; centrifugal forces which prevent the two nuclei to fuse into a single composite system and to move towards a spherical shape. A schematical one dimensional representation which might nevertheless be useful is to consider a potential energy as a function of the distance between the two centers like in molecular physics, and to keep all degrees of freedom frozen, except the relative motion. The potential energy reserves the structure of each nucleus during the contact. Such a "sudden approximation" has received a justification by Seglie et al.[38], based on the existence of a strong friction in the entrance channel. Of course, when the critical distance, i.e., a sufficient overlap of nuclear densities, is reached, one should unfreeze other degrees of freedom and abandon the two body potential in order to proceed to an attractive

potential well representing the compound nucleus. At the "point of no return", the two nuclei stick together due to the loss of energy large enough to establish a common nuclear structure.
The Coulomb potential for $r < R_1 + R_2$ is calculated according

$$V_c = \frac{3e^2}{5}\left(\frac{(Z_1+Z_2)^2}{(R_1^3+R_2^3)^{1/3}} - \frac{Z_1^2}{R_1} - \frac{Z_2^2}{R_2}\right) - kr^n \qquad (14)$$

where k and n are adjusted in order to fit

$V = \frac{Z_1 Z_2 e^2}{r}$ for $r = R_1 + R_2$.

The centrifugal potential is taken as $\frac{\ell(\ell+1)\hbar^2}{2\mu r^2}$,

but the moment of inertia should be changed into a larger value than μr^2 at the sticking point when the entire system rotates.

The nuclear potential was calculated first by Galin et al. [33] using the energy density formalism. More sophistacated calculations were made by Ngô [22] using Hartree-Foch BCS densities. Ngô et al. [35] have shown that they can write the potential as the product of a "universal function" and of a geometrical factor, connected with the proximity theorem of Randrup, Swiatecki and Tsang [39].

The critical potential V_{cr} is obtained for $\ell = 0$, from $V_{(R)} = V_{coul} + V_N$ at the distance R_{cr}. From the analysis of a great number of experimental results, a critical radius parameter

$$r_{cr} = R_{cr}/(A_1^{1/3} + A_2^{1/3})$$

was deduced and was found equal to 1 ± 0.07 fm.

An interesting observation is that at such a distance, the potential $V(R) = V_{coul} + V_N$ for $\ell = 0$ shows nearly always its minimum value, so that $V_{cr} = V_{(R_{cr})}$ might be directly extracted when $\frac{dV}{dR} = 0$ as shown on figure 10.

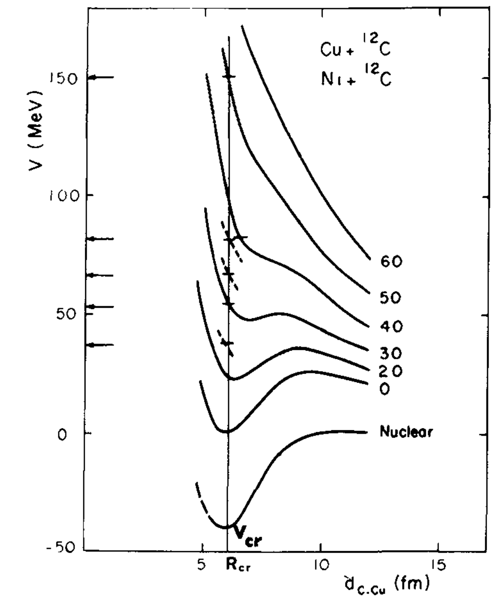

Fig. 10 - Calculated potential energy curves for the system ($Ni+^{12}C$) at various $\ell\hbar$ values (ref. [33]. Experimental critical ℓ values were obtained at various energies by Namboodiri et al [41]. The critical potential is shown, close to the bottom of the nuclear potential, at $R_{cr} = (Z_1Z_2/A_1^{1/3} + A_2^{1/3})$, i.e. for $r_{cr} = 1$ fm.

Ngô [40] has found that V_{cr} and the product Z_1Z_2 are related empirically by a nearly linear expression that may be very useful.
For $r_{cr} = 1.0$ fm, and $Z_1Z_2 < 1000$

$$V_{cr} = (0.124275\ Z_1Z_2 - 17.6)\ \text{MEV}$$

and for $Z_1Z_2 \gtrsim 1000$, $V_{cr} = (0.11705\ Z_1Z_2 - 6.9)$ MEV

The model of Bass [34] is the simplest analytical expression of a series of one dimensional models formulated for spherical nuclei based on the liquid drop concept. He has used a nuclear potential inspired by the Krappe and Nix's approach [23],

$$V_N(R) = \frac{a_s A_1^{1/3} A_2^{1/3} d}{R_{12}} \exp\left(-\frac{r-R_{12}}{d}\right) \qquad (15)$$

where R_{12} is the characteristic distance for fusion between the two centers analogous to the critical distance R_{cr}. It is taken as r_{cr} ($A_1 + A_2$) with $r_{cr} = 1.07$ fm, R_{12} is essentially equal to the sum of the half density radii.
Then ℓ_{cr}^2 (or $\sigma_{CF}/\pi\lambda^2$)
is obtained by solving the equation

$$\bar{E} = \frac{Z_1 Z_2 e^2}{R_{12}} + \frac{l_{cr}^2 \hbar^2}{2\mu R_{12}^2} - \frac{a_s A_1^{1/3} A_2^{1/3} d}{R_{12}} \qquad (16)$$

(where d the diffuseness parameter is equal to 1.35 fm and a_s, the surface energy constant=17 MEV)

In this concept, the two densities are allowed to undergo gradual deformation. Although it is not completely an adiabatic process, it differs from the energy density formalism by rearrangement effects as the two nuclei overlap and then there is a deeper potential Well.

According to Bass, complete fusion should not be possible when $\frac{dV}{dR}$ stays always negative, i.e., when the potential at R_{12} cesses to be attractive. This occurs when the centrifugal potential is too high precisely at an energy given by :

$$\bar{E}_{sat} = \frac{Z_1 Z_2 e^2}{R_{12}} + \frac{1}{f^2}\left(\frac{a_s A_1^{1/3} A_2^{1/3}}{2} - \frac{Z_1 Z_2 e^2}{2R_{12}}\right) - \frac{a_s d A_1^{1/3} A_2^{1/3}}{R_{12}} \qquad (17)$$

The factor f comes from the asumption that the angular momentum decreases at the point of contact in a model of Complete Sticking and rigid rotation as a whole. Then, applying Huygens theorem for the moment of inertia,

$$\frac{1}{f} = 1 + \frac{2}{5}\left(\frac{A_1 R_1^2}{\mu R_{Fus}^2} + \frac{A_2 R_2^2}{\mu R_{Fus}^2}\right) \qquad (18)$$

depends on the mass ratio A_1/A_2 and is equal to 7/4 for equal masses.

In this model ℓ_{cr}^2 increases linearly with the energy up to a saturation value at which it becomes constant.

Such a restriction does not apply to the critical distance and critical potential model. In such model there is no necessity for a well in the potential curve for complete fusion, since it is based on a sudden approximation hypothesis. The only criterium to be fulfilled is that the intersect between the kinetic energy line and the potential curve at a given ℓ value, should occur at a distance closer than R_{cr}. If so, a fusion zone is reached where the two body description should be abandonned and there is an irreversible drift towards a single potential well.

Both models fit remarkabely well the experimental results on σ_{CF} as far as the energy does not exceed the saturation value calculated by Bass. This is shown in figure 11 extracted from a recent publication of Namboodiri et al.[41], where Bass's calculations and a number of experimental data are compared.

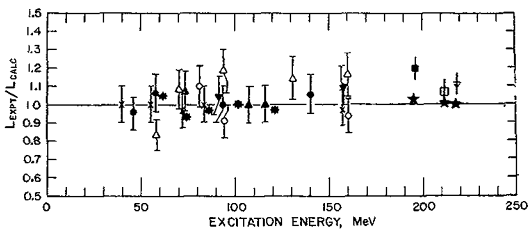

Fig. 11 - Comparison of experimental and calculated critical ℓ values made by Namboodiri et al [41]. Calculations are made according Bass' model [34] with "saturation".
Stars correspond to the critical distance's model [33] calculation at the highest energies (see text).
Asterisks are the new data by Scobel et al [16].

A number of new points have been added, mainly on the systems ^{35}Cl + ^{27}Al, ^{35}Cl + ^{48}Ti, ^{35}Cl + ^{62}Ni, ^{35}Cl + ^{90}Zr, ^{35}Cl + ^{116}Sn and ^{35}Cl + ^{124}Sn recently studied by Scobel et al. [16] at energies high enough to induce a strong difference between σ_{CF} and σ_R .

The treatment of the same systems by the critical distance of approach and a correct determin ation of V_{cr} gives an even better agreement, as it has been shown in a recent report [32] for most of the available data, and as it appears in the following table for the ^{35}Cl induced reactions.

Table II

Target	Energy c.m. (MeV)	σ_{CF} (mb) ER+Fis	V_{cr} (MeV) at 1 fm	$\sigma_{calc} = \pi R_{cr}^2\left(\frac{\bar{E}-V_{cr}}{\bar{E}}\right)$
^{27}Al	69.4	1140	9.9	1030
^{27}Al	73.8	1200	9.9	1065
^{48}Ti	92.2	1080	28.9	1030
^{56}Fe	98.1	964	37.3	980
^{58}Ni	87.1	781	41.6	830
^{62}Ni	108.4	1091	41.6	1010
^{90}Zr	118.4	850	66.9	820
^{116}Sn	130.1	560	88.0	650
^{124}Sn	132.3	632	88.0	700

Recently, Namboodiri et al. [41] have made experiments with 262 MEV nitrogen ions on ^{27}Al, Ge and Ni target nuclei. Energies in the center of mass exceed the saturation value of the Bass model expressed in [17]. Complete fusion cross section measurements indicate that the predicted saturation angular momenta are exceeded and therefore the ratio $\frac{L_{exp}}{L_{calc}}$ of limiting values becomes higher than 1, as shown on figure 12.

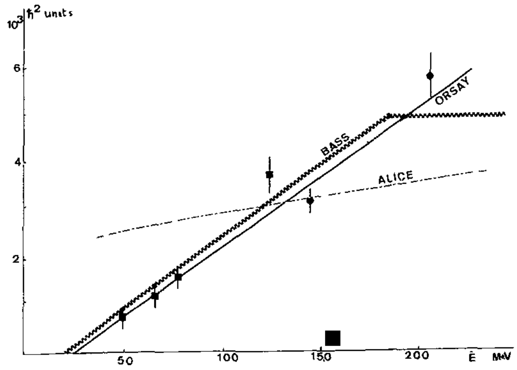

Fig. 12 – Plot of $\sigma/\pi\lambda^2 = \ell^2_{cr}$ in $\hbar^2$ units versus center of mass energy, $\bar{E}$. Squares : experimental results for $^{12}C+^{63}Cu$. Circles : experiments for $^{14}N+Ni$. Calculated values are indicated for three models : Bass [34], Ngô [40], and ALICE code [42]. Notice the saturation effect in Bass' model is not followed.

The authors have admitted that above the saturation energy, the limiting angular momentum may be calculated whithout taking account of the saturation and then there is a good fit with experimental data. However such a procedure is not entirely justified in the frame of a liquid drop model where the fusion conditions should indeed be connected with a potential well.

On the contrary, the energy density model still shows an excellent agreement with the new experimental data. The difference at high energy between the previsions of both model is shown on figure 12 which has been drawn for $^{12}C + ^{63}Cu$ and $^{14}N + ^{58}Ni$, two systems that present nearly the same calculated features.

Another aspect which is raised by these new results [41] from Natowitz's group is the comparison with the calculations of Plasil and Blann [42] based on the angular momentum dependent fission barriers of a rotating liquid drop. I shall discuss later on a bit more the conclusions of this concept and of its incorporation into the evaporation code ALICE. Let us remind for the moment that, starting with the computation of Cohen Plasil and Swiatecki [43] on the lowering of fission barriers B_f by the centrifugal effect, Blann and Plasil admit that when $B_f(J)$ disappears, the corresponding angular momentum is the critical value in the entrance channel.

Namboodiri et al. have shown that the production of non-fissioning compound nuclei corresponds to angular momenta which exceed the critical limit using the liquid drop model. Furthermore, although the ALICE code predicts large cross sections for symmetric fission, they were not observed. Therefore, the $B_f(J) = 0$ concept does not appear to provide an appropriate description of the mechanism limiting σ_{ER} at least in compound systems lighter than ^{72}Br.

Expressions [4] and [13] are the two asymptotic forms of a more general expression of σ_{CF} , connected with two distances for fusion, the first one close to the barrier R_{Fus} where the difference between σ_{CF} and σ_R is only due to the tail of nuclear matter, and the second due to dynamical effects and expresssed as $R_{cr} = 1.0\ (A_1^{1/3} + A_2^{1/3})$ fm.

Therefore, the presentation of σ_{CF} versus $1/\bar{E}$ should exhibit two straight lines. This is illustrated very nicely in three figures.

For light systems, the first of these figures (fig.7) has already be showned [27] for ($^{12}C + ^{16}O$).

A second one is due to Lee et al. [44] for the system $^{14}N + ^{12}C$ (fig. 13).

The third one illustrates [45] a much heavier system ($^{40}Ar + ^{121}Sb$) (fig. 14).

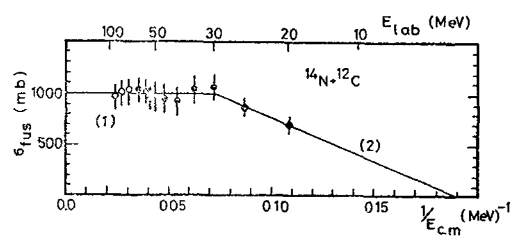

Fig. 13 – Plot of σ_{CF} versus $1/\bar{E}$ for the system $^{14}N+^{12}C$, by Lee et al [44]. Experimental points at low energies are due to Volant et al [46].

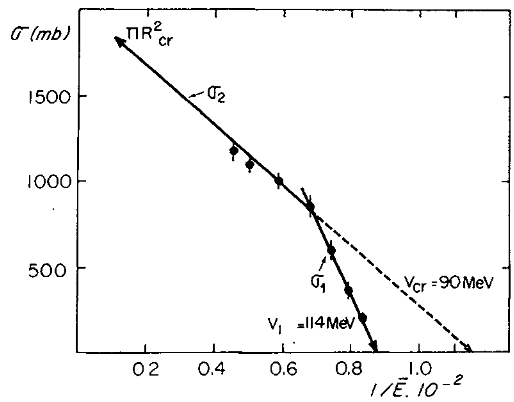

Fig. 14 - Plot of σ_{CF} versus $1/\overline{E}$ for the system $^{40}Ar+^{121}Sb$, by Gauvin et al [45]. σ_1 corresponds to complete fusion without critical value limitation. V_1 is the potential at $dV/dR = 0$ for $\ell = 0$. σ_2 corresponds to expression (13) and $V_{cr} = 90$ MeV and $r_{cr} = 0.98$ fm.

V_{cr} and R_{cr} could be extracted and were compared to Ngo's calculated values in table III. The agreement was good for Argon induced reactions and also for the system $^{12}C + ^{18}O$. Although the system ($^{12}C+^{14}N$) should indicate a negative V_{cr} because of the low Coulomb potential, the experimental curve gives $V_{cr} = 0$, and the critical radius is also much higher than expected. This is an unexplained disagreement, although one may suggest, as it has been done by Volant et al. [46] that the limit is, for that particular case, not ruled by the entrance channel dynamics, but is given by the compound nucleus ^{26}Al Yrast levels.

Table III

Comparison of experimental and calculated V_{cr}

System	V_{cr} (MeV)	r_{cr} (fm)	V_{cr}. calc at 1 fm
$^{12}C+^{18}O$	- 14	1.01	- 11.6
$^{40}Ar+^{121}Sb$	+ 90	0.98	+ 96
$^{12}C + ^{14}N$	0	1.2	- 12.4

Concerning this last point, it has been demonstrated many times that for medium and heavy nuclei the limitation is not due to the properties of the compound nucleus, mainly to the existence of Yrast levels. Crucial experiments have shown [47] that making the same compound nucleus at the same excitation energy but through different combinations does not give the same critical angular momentum. However, for light systems like ^{24}Mg, this might not be entirely true. In that respect, one should also mention a new result [48] obtained by comparing the systems ($^{16}O + ^{63}Cu$) and ($^{34}S + ^{45}Sc$) which lead to the same ^{79}Rb compound nucleus, indicating that, in a range of energies well above the interaction barrier, one can deduce from excitation functions the same critical angular momentum. However the way the results have been extracted was not the measurement of complete fusion cross sections, and it is worthwhile to spent some time on this interesting method and to discuss its validity.

3.3. Determination of $l_{cr}\hbar$ from the shape of excitation functions for final residual nuclei. Spin dependent evaporation calculations.

The fact that heavy ions introduce large angular momenta in the compound nuclei has a profound effect on excitation functions. If one considers a reaction in which x particles are emitted, for example 4 neutrons, the corresponding final residual nucleus is produced in a certain range of excitation energies. The very large angular momentum population broadens the range for a given excitation function. If the compound nucleus shares a large angular momentum, an important fraction of the energy is dissipated by gamma rays, particularly when the Yrast line region is attained. Therefore a given excitation energy will "evaporate" a smaller number of particles than for a low angular momentum compound nucleus. Moreover, the emitted particles have a broader kinetic energy spectrum, and then a larger average kinetic energy, and the result will reduce the number of emitted particules for a given excitation energy. The excitation function for a given product is a sum of excitation functions for formation of this product from compound nuclei of different angular momenta. Since in principle, all the angular momentum population exists from J = 0 until some maximum limit, the threshold of the excitation function is at the same energy as for reactions induced by light particles. But the maximum cross section is shifted at higher energy. Such an effect has been calculated quantitatively by Kammuri etal. [49] in 1963 ; in a comparison of the reactions $^{65}Cu(p,3n)\,^{63}Zn$ and $^{51}V(^{14}N,3n)\,^{62}Zn$ which proceed nearly through the same compound nucleus (^{66}Zn or ^{65}Zn). The maximum cross section is 10 MeV higher for the ^{14}N induced reaction than for the p induced reaction. Most experiments have shown such results in accord with these expectations. For neutron-evaporation reactions induced by protons and alpha particles, the excitation functions have a full width at half-maximum of the order of 10-15 MeV. There are many examples of that number and a few of them are quoted here in the region of medium masses. The reaction $^{169}Tm(p,3n)$ has an excitation function [50] with FWHM = 10 MeV. For the reaction $^{133}Cs(p,3n)$ ^{131}Ba, the excitation function [51] has a FWHM of 12 MeV ; while reactions induced by ^{14}N, ^{16}O, ^{20}Ne and ^{40}Ar in the same region compound nuclei produce FWHM values between 20 and 45 MeV, as shown on figure 15.

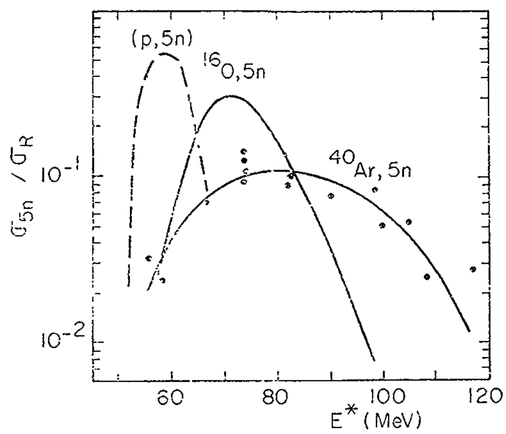

Fig. 15 - Excitation functions for 5 neutrons emitted by a compound nucleus ^{158}Er produced by proton, ^{16}O and ^{40}Ar projectiles. Experimental points are shown for the (^{40}Ar,5n) reaction.

The idea that a quantitative analysis of the width and particularly a good reproduction of the slope on the high energy side of the excitation function could be used in order to estimate the J population, was first introduced in 1974 at the Enrico Fermi School [52] . Figure 16 extracted from these lectures shows an example of reproduction of experimental points by imposing an upper limit for the reaction $^{118}Sn(Ar,5n)^{153}Er$. The first attempt was based on the crude asumption that the excitation energy may be divided into intrinsic and rotational energies, and the rotational energy evaluated as

$$E^{*}_{rot}(J) = \frac{J^2\,\hbar^2}{2\,\mathcal{J}_{rig}}$$

is assumed to be unavailable for particle emission. Then the level density:

$$(E^{*},J) = \rho(E^{*} - E_{R\,rot}(J))$$

and the classical statistical evaporation theory can be applied. Since that first trial, a more sophisticated code has become available, made by Blann and Plasil [42] and named ALICE. This code is very widely used for predicting evaporation residue cross sections and is indeed extremely useful for crude estimations.

However, it should be recalled that expressing the level density for states of spin J at excitation energy E^{*} as the level density for all states at an excitation energy lowered by the quantity $J^2\,\hbar^2/\,2\,\mathcal{J}rig$ comes from a number of crude approximations :

i) a Gaussian distribution of angular momentum projections,

ii) the Fermigas model,

iii) and an expansion to the first order of exponcutial terms in the expression of ρ (E^{*},J).

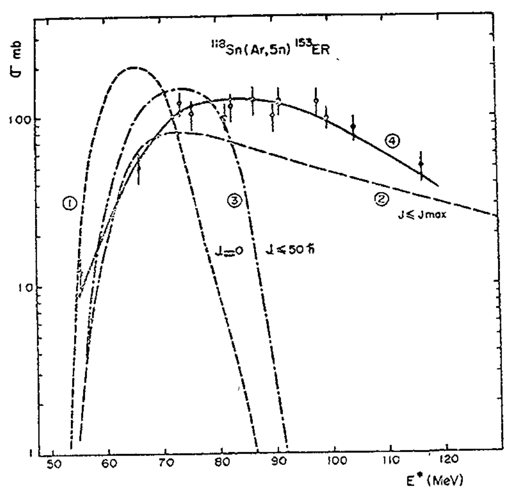

Fig. 16 - Calculation of the excitation function for the reaction $^{118}Sn(Ar,5n)$: (1) without spin , (2) with J population up to $J\hbar = \ell\hbar_{max}$, (3) with a critical value J = 50, and (4) with J_{cr} varying from 72 to 80 ([52,76])

For a number of nuclei, the level density on the right side of the well known plane (E^*,J), in the vicinity of the Yrast line, might be quite different from the level density on the left side, close to J = 0, even at lower E^* values. This is well demonstrated by a more elaborated calculation made by Fleury et al. [53] with a correct evaluation of level densities for some particular nuclei.

Figure 17 taken from these authors shows very clearly that α emission is the main decay process from ^{188}Pt excited at 60 MeV with 60 < J < 70, while at E^* = 30 MeV and J = 0 ($E^* - E_{rot}$(J=60)), neutron emission dominates entirely. On the contrary, the method might be applicable to a nucleus like ^{113}Te where neutron emission dominates over all the J population at excitation energies higher than 30 MeV.

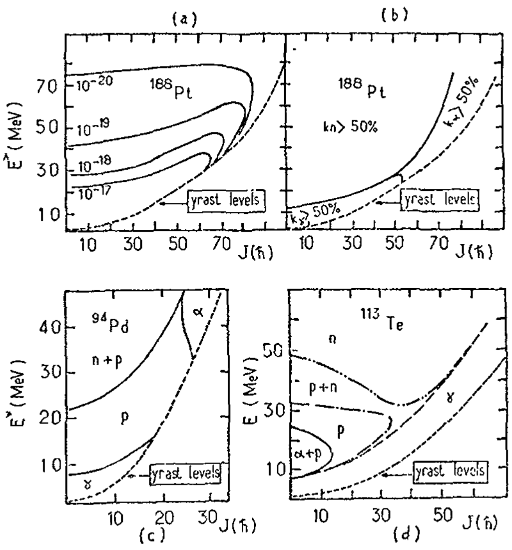

Fig. 17 - Contour plots in the plane E^* versus $J\hbar$, for the compound nuclei ^{188}Pt, ^{94}Pd and ^{113}Te, according to calculations made by Delagrange et al [53]:a) Life time, b), c) and d) probabilities higher than 0.5 for emission of neutrons, protons, α particles and γ radiation.

Another point in the Alice code is related to the use of the rotating liquid drop fission barrier, for estimating the limiting value of the complete fusion cross section, but we shall discuss it later on.

Now, coming back to the analysis of an excitation function for a residual nucleus produced after only neutron emission, we want to stress that the limiting value of J deduced from the application of the code Alice might not represent the actual critical value for complete fusion . Taking the example of ^{94}Pd in Fleury's picture, it would correspond only to that range of J where neutron and proton emission dominates, while compound nuclei with higher J decay exclusively after α decay and residual nuclei for that zone are at least 2 Z units lower than the compound system (or 4 Z units lower if two α are emitted in cascade). Therefore one should be very cautious about the conclusions, if only one category of excitation functions have been studied, as it has been the case for the system ^{79}Rb, by Langevin et al.[48]. On the other hand, the excitation function is very sensitive to the limiting J value, and the method is indeed more precise that cross section measurements, as shown on figure 18. Then it is a very useful one.

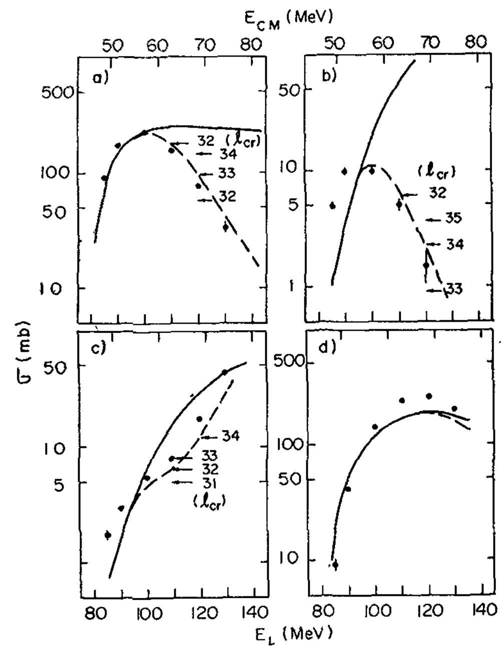

Fig. 18 - Determinations of ℓ_{cr} values on the systems $^{34}S+^{45}Sc$ by Langevin et al [48]. Excitation functions correspond to the following excitation channels : a) 2pn and 2np, b) 2p,np and 2n, c) αn and 2p3n, d) 3np, 2p2n and 3pn. The lines correspond to the code ALICE (solid lines - all partial waves, dashed lines - indicated ℓ_{cr} values).

The best would be to check that the limit is the same for (HI,xn) and for (HI, αxn) excitation functions, or to verify that the total evaporation residue cross section is in agreement with ℓ_{cr} deduced from excitation functions. In my opinion, there is no special reason for the moment to believe that α particles are emitted [48] in a direct process before complete fusion of ^{34}S and ^{45}Sc, and evacuate a certain amount of angular momentum in order to leave a nucleus with a lower J value, as far as the decreasing branch of an (HI,αxn) reaction has not been yet analysed.

4. Limitation to Complete fusion for heavy and very heavy nuclei. Is there a distinction between fission after complete fusion and pre-equilibrium fission or quasi-fission ?

When we move to composite systems above the rare earth region, and to massive projectiles bringing large orbital angular momenta, fission process becomes indeed an important contributor to the decay of a rotating compound nucleus. Then the cross section for evaporation residues decreases very rapidly as the product Z_1Z_2 increases. In order to obtain σ_{CF}, one should measure both σ_{ER} and σ_{fiss}. One difficulty is to define specifically fission events coming from a truly excited compound nucleus and we have already mentioned this problem in the introduction.

As a matter of fact, the evaporation residue cross section diminishes in the benefit of two processes, the first being fission competition in the de-excitation of the compound nucleus, the second being Deep inelastic or quasi-fission phenomena which involve a large proportion of incident ℓ waves when both energy and orbital angular momenta increase. Let us present, in figure 19, some recent results as a function of $\frac{Z_1Z_2}{A_1+A_2}$ which is similar to the fissility parameter $\frac{Z^2}{A}$.

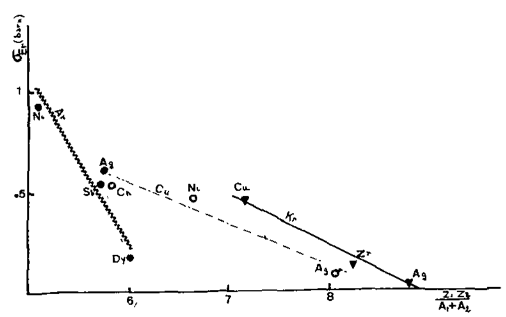

Fig. 19 - Cross section for evaporation residues as a function of the parameter $Z_1Z_2/(A_1+A_2)$ in argon induced reactions on Ni, Sb, Ag and Dy [45], copper induced reactions on Ni and Ag, chromium on iron [54], and krypton on Cu, Zr and Ag [55].

They have been obtained at different center of mass bombarding energies and the comparison is not entirely satisfying. However, figure 20, due to Gauvin et al. [54] shows' that the cross section is no very dependent on energy, although σ_{ER} / σ_R decreases rather drastically, as one should expect if the highest ℓ waves contribute to other processes than compound nucleus formation.

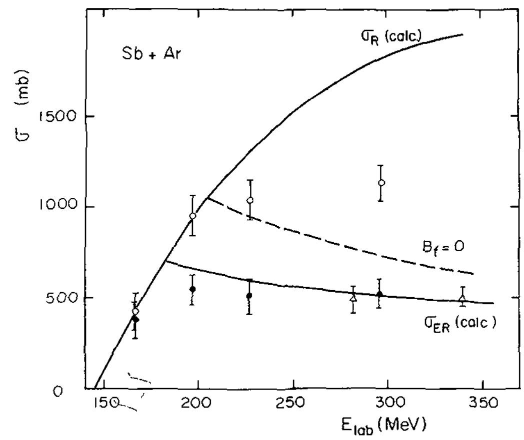

Fig. 20 - Cross sections for evaporation residues in the reaction (^{40}Ar+Sb).
Dark points : experimental results [45].
Open circles : $\sigma_{CF} = \sigma_{ER}+\sigma_{fiss}$.
σ_{ER} calculated has been obtained with the ALICE code.
The dashed curve with $B_f = 0$ is the calculated σ_{CF} using the rotating liquid drop fission barrier as a limit for $\ell_{cr}\hbar$.

4.1 - Fission reactions induced by heavy ions in the mass region 100-200.

There is a general agreement that all fission fragments observed in carbon or oxygen induced reactions on targets in the rare earth region originate from the fission of de-exciting compound nuclei.

The fission cross sections in that region is a relatively small fraction of the total reaction cross section and angular correlation experiments have shown that full momentum transfer has occured. In addition, fragment kinetic energies (Viola and Sikkeland [56]) (Plasil et al [57]) and mass distributions have been measured and interpreted in term of the de-excitation process of fully equilibrated compound nuclei. Furthermore, angular distributions measured for fissions of rare earth compound nuclei behave like $1/\sin\theta$ in the center of mass system, indicating that the composite system has survived sufficiently long to undergo one or more full rotations (Gordon et al [58]) (Zebelman et al [47]). The $1/\sin\theta$ shape can be qualitatively understood if one reminds that when the deformation leading to fission occurs along an axis in the rotational plane the dumbbell configuration rotates about a perpendicular axis and splitting is highly favoured. By integrating differential cross sections over the $1/\sin\theta$ angular distribution, cross sections for heavy ion induced fission have been obtained. For example, Sikkeland [50] found that, in the reaction $^{22}Ne + {}^{159}Tb$, the compound nucleus ^{181}Re has a probability for fissionning of only 2.10^{-3} at an excitation energy of 60 MeV while it increases until nearly 80 % at 120 MeV. More recently, Zebelman et al [47] have found a fission cross section of around 100 mb in the reaction ($^{20}Ne + {}^{150}Nd$) at 107 MeV of excitation energy while for the same compound nucleus ^{170}Yb, at the same excitation energy, the reaction $^{12}C + {}^{158}Gd$ yields only a few mb of fission.

The effect of angular momentum on the fission probability has been extensively described by Plasil at the Nashville Conference [9]. Let us remind that the competition between neutron emission and fission can be expressed crudely as

$$\Gamma_n/\Gamma_f = C \exp\left(-\frac{S_n - B_{fR}}{\tau}\right) \qquad (19)$$

where S_n is the neutron binding energy, τ the nuclear temperature and B_{fR} the rotating liquid drop barrier defined as

$$B_{fR} = B_f - (E_{Ro} - E_{Rs})$$

where B_f is the fission barrier, $E_{Ro} - E_{Rs}$ the difference in rotational energies in the spherical form E_{Ro} and at the saddle point E_{Rs}.

For very heavy targets ($A > 200$), the totality of the compound nuclei undergoes fission, since B_f is already lower than 15 MeV and B_{fR} can easily decrease at a value lower than S_n, even for medium ℓ values. Therefore, the complete fusion cross section should be very close to the fission cross section and Sikkeland has proposed to obtain σ_{CF} by the measurement of the fission fragment cross section in the case of $^{16}O + {}^{238}U$ and $^{20}Ne + {}^{238}U$. The first care that should be taken is of course to insure that fission fragments are not originated from an excited nucleus in the vicinity of uranium issued from a transfer reaction. This has been checked (Sikkeland et al. [8]) by fixing one detector and varying the angle of a second detector. Counting the fragments in coincidences, a peak was obtained at a kinematic angle that corresponds to full momentum transfer. A second peak or a shoulder was observed in the angular correlation, consistent with momentum transferred by ^4He ions. It was assumed that the main peak was due to the fission of the compound nucleus. In the reaction $^{20}Ne + {}^{238}U$, the composite system was ^{258}No, and it has been argued that all degrees of freedom were perhaps not entirely equilibrated before fission took place. However, it seems very difficult to distinguish a pre-equilibrium fission from a compound nucleus fission. If the composite nucleus has time to equilibrate in all degrees of freedom that determine charge and mass distributions an intrinsic excitation energy, then we should observe all the characteristics of normal fission. As a matter of fact, there is probably a continuous transition between direct "fission" where masses are not yet equilibrated and where the mass distribution is still keeping the memory of the entrance channel, and a totally equilibrated phenomenon where all degrees of freedom have been involved in the statistical equilibration process. For the moment, as far as angular momenta ar lower than 100 $\hbar$, and compound nuclei are known to be bound, it seems reasonable to believe that complete fusion cross sections have been correctly measured up to compound nuclei like ^{258}No.Table IV gives cross sections and the deduced ℓ_{cr} values, when fission cross sections are included.

4.2. Distinction between fission after complete fusion and "direct fission" induced by medium mass projectiles.

For argon induced reactions, large orbital angular momenta are obtained even at moderate kinetic energies and B_{fR} is so much lowered that the fission cross section is not negligeable as it was shown [60] in the reaction (Ar + Sb). For systems having mass numbers less than about 100, it is predicted that the fragment mass distribution is no longer peaked at symmetric division (Businaro - Gallone [62]) and it becomes nearly impossible, if it is so, to distinguish between fission and deep inelastic transfer reactions. Therefore the cross sections for evaporation residues which have been measured are certainly lower than σ_{CF}, but we don't know of what amount exactly.

The second point concerning Ar induced reactions is relative to very heavy nuclei. Since very large angular momenta are brought to the composite system, and also since such a composite system in some cases is not at all known, the questions raised previously on the problem of "direct fission" against "full equilibrated fission" become indeed serious.

However, the reaction Ar + Au has been studied with a great detail of care, at 1.2 time the barrier.

First, it has been shown by Tamain et al.[60] that a large number of fragments present a symmetric mass distribution around $(A_1+A_2)/2$.

Second, Ouichaoui et al. [63] at this Conference show on figure 21 that there is a welldefined difference between this mass distribution and the light quasi-fission fragments.

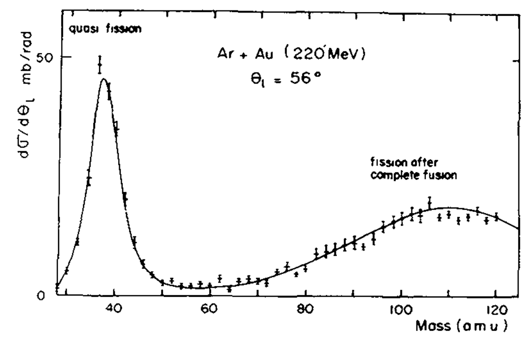

Fig. 21 - Mass distribution for the fragments produced in the reaction $^{40}Ar+^{197}Au$ at 220 MeV. There is a very clear distinction between quasi-fission or deep inelastic light fragments around A = 40,,and fission fragments [63].

Third, the angular distribution has been studied by collecting emitted fragments on catcher foils and counting X rays (Lucas et al.[64]). It has been shown that all elements between Z = 40 and Z = 65 (the compound nucleus is Z = 97) exhibit an angular distribution in $1/\sin\theta$ ($\frac{d\sigma}{d\theta}$ = cst), while elements with $70 < Z < 83$ present a peak around $\theta_{cm} = 120°$, a distribution quite in agreement with a quasi - fission or DIC process.

The conclusion is that more than 50 % of the cross section correspond to complete fusion nuclei which live a time longer than the rotation period (10^{-21} sec.), even though these nuclei have no fission barrier. The composite system which ends up into strongly damped products (quasi-fission) live much smaller time (around 10^{-21}sec.). In my opinion these results are a good demonstration that the fission barrier concept is valuable in the exit channel along the path to fission, but should not be used as a criterion for the limit for complete fusion in the entrance channel because the collision follows another path.

One might still argue that there is not a full equilibration in the compound nucleus. Nevertheless one is entitled to call the phenomenon complete fusion, as in table IV. (See following page)
The result is certainly very different from the observation made with heavier ions like ^{63}Cu and ^{84}Kr, which consists of fission-like fragments for the kinetic energy, while the mass distribution is not at all symmetric around half mass of the composite system and still reflects the projectile and target masses. Here we move to a well defined non-compound fission phenomenon and the distinction seems rather clear between complete fusion and fissionas observed with Ar ions and a faster process shorter in time than one revolution that has been called "quasi - fission" (Hanappe et al. [72]).

4.3 - Limitation to Complete Fusion Contribution from low ℓ waves.
Very heavy Projectiles.

For projectiles heavier than A = 40, presently, Fe, Cu and Kr ions, a number of experimental results indicate clearly that an additional limitation occurs for complete fusion and lowers σ_{ER} as well as σ_{fiss}. There are mainly two categories of results concerning the interaction of very heavy ions with complex nuclei.

The first experimental evidence for negligeable complete fusion followed by full momentum fission was given [70] on the accelerator ALICE. In dis - agreement with the announcement made by Oganessyan [71] that a wide mass distribution of fission fragments were observed in the bombardment of Ta by Kr ions, it was demonstrated that in the interaction of 500 MeV Krypton ions with ^{238}U and ^{209}Bi nuclei, no symmetric fission issued from a compound nucleus was observable and an upper limit of 30 mb was indicated. Furthermore, large cross sections for symmetric fissions following full momentum transfer were

Table IV

Complete fusion cross sections ($\sigma_{ER} + \sigma_{fiss}$)

Projectile	Target	CN	$\bar{E}$ (MeV)	E^* MeV	σ_{ER} (mb)	σ_{fiss} (mb)	σ_{CF} (mb)	l_{cr}	l_{max}	Ref
^{40}Ar	^{77}Se	^{117}Te	95	71				(52)	70	65
^{40}Ar	^{77}Se	^{117}Te	132	107				(70)	110	65
^{40}Ar	^{74}Ge	^{114}Sn	128	110	950		>950	>69	90	54
^{40}Ar	^{109}Ag	^{149}Tb	144	92	950		>950	>75	80	19
^{40}Ar	^{109}Ag	^{149}Tb	210	158	600	650	1250	113	140	19
^{32}S	^{115}In	^{147}Tb	269	210	530	330	860	92	110	69
^{40}Ar	^{121}Sb	^{161}Tm	148	84	550	500	1050	84	91	54
"	"	"	170	108	520	530	1040	89	111	54
"	"	"	222	161	520	610	1030	107	155	54
^{20}Ne	^{150}Nd	^{170}Yb	127	107	1450	100	1550	73	80	47
^{40}Ar	^{165}Ho	^{205}At	242	155	<10	1350	1350	126	163	60
"	"	"	182	95		800	800	84	108	60
^{12}C	^{197}Au	^{209}At	81	63,5	560	600	1160	42	50	66
^{12}C	^{238}U	^{250}Cf	115	89,5		1550	1550	56	72	67
^{16}O	^{238}U	^{254}Fm	154	115		1490	1490	73	88	67
^{20}Ne	^{238}U	^{258}No	185	134		1360	1360	86	114	67
^{40}Ar	^{238}U	$^{278}110$	214	82		516	516	76	116	60
^{40}Ar	^{238}U	$^{278}110$	257	125		1030	1030	117	164	60
^{40}Ar	^{238}U	$^{278}110$	342	204		1330	1330	150	230	68

measured in the reaction ($^{40}Ar + {}^{238}U$) while the cross section was very low for ($^{84}Kr + {}^{186}W$) although $Z_1 + Z_2 = 110$ is obtained for both composite system. It is known, now a day, that, instead of leading to complete fusion, even low ℓ waves contribute to a very striking process which was named "quasi - fission" [72] and has the same characteristics as the Deep inelastic collisions that shall be described in detail by J.Galin [12] and L.Moretto [13], at this Conference. It has been confirmed in Berkeley [73] by radiochemical analysis that the wide mass distribution announced by the Dubna Group did not correspond to a symmetric fission.

Between Argon induced reactions and Krypton induced reactions, an interesting intermediate situation has been studied by Pēter et al. [74] with copper projectiles. Bombarding Au at an energy very close to the barrier (1.1 $\bar{B}$), the amount of fission following complete fusion is very small, and nearly all the cross sections go into deep inelastic collisions, amongst which 55 % correspond to quasi - fission events (totally relaxed) and 50 % to partially damped processes. At a higher energy (1.4 B) a number of events can be interpreted as symmetric fission following complete fusion, and the cross section is of the order of 100 mb. These result show that there is a great difficulty for complete fusion, even in head-on collisions, and one has to increase the energy well above the interaction barrier in order to begin to initiate this process.

The second set of results showing how complete fusion becomes difficult with very heavy projectiles (A > 40) concerns a detailed study of evaporation residue formation as a result of ^{158}Er compound nucleus decay, and a comparison of the results when different combinations of projectiles and targets were used, namely :
($^{16}O + {}^{142}Nd$), ($^{40}Ar + {}^{118}Sn$), ($^{84}Kr + {}^{74}Ge$) and ($^{63}Cu + {}^{96}Zr$), the last one giving a slightly different compound system, ^{159}Tm.

Compound nuclei were formed without ambiguity since evaporation residues were recognized [75-76] There was certainly a fraction of the complete fusion cross section that undergoes fission, but we shall focuse our attention on excitation functions of residual nuclei resulting from (Kr,xn) or (Cu,xn) reactions, i.e. obtained as decay products after the evaporation of x = 3, 4, 5, 6 neutrons. Let us take the cross section for ^{153}Er which results from the compound nucleus ^{158}Er after evaporation of 5 neutrons (Figure 22).

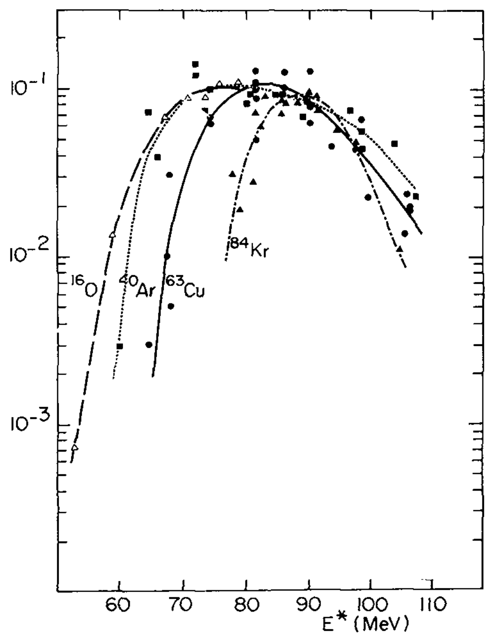

Fig. 22 - Excitation functions for (HI,5n) reactions passing through the same compound nucleus ^{158}Er (^{159}Tm for ^{63}Cu projectiles). The shift between ^{16}O and ^{84}Kr is clearly seen [75-76].

The threshold was observed at an excitation energy 15 MeV above the threshold obtained for the same de-excitation process (5 neutrons from ^{158}Er) but when the projectile was ^{16}O and the target ^{142}Nd. A third system passes through the same compound nucleus (^{40}Ar + ^{118}Sn) and whereas there is a small shift towards higher energies when going from ^{16}O to ^{40}Ar, the shift is very large between Argon induced 5n reaction and Krypton induced 5n reaction. The same thing was observed for x = 6 and x = 4. The absolute magnitude of the cross sections was smaller about a factor two in the case of Kr and the excitation function was not only shifted but also narrower, as shown on figure 22.

How to explain these unexpected results ? In principle, all (HI,5n) excitation functions should start at the same excitation energy whatever is the heavy ion since the threshold corresponds to the sum of binding energies for 5 successive neutrons emitted from ^{158}Er. This is indeed, observed when comparing (C,5n), (O,5n), (Ne,5n) reactions. The low excitation energy part of the curve should be attribued to the low angular momentum population, since all the available energy has to be taken for the evaporation of 5 neutrons. If some energy was dissipated by gamma rays as it is when the compound nucleus shares a large J$\hbar$, there would not be enough energy left for emitting 5 neutrons and the resulting nucleus would be ^{154}Er. In principle, only the maximum of the excitation function and the descent on the high energy side should be shifted towards higher values and the FWHM should be increased by the effect of large J in the angular momentum population. A quantitative treatment [76], similar to the code Alice, on the same basis as was explained in section 3.3, has been made in order to fit excitation functions. For oxygen induced reactions, all available J values were taken. For Argon projectiles, a cut-off was necessary on the high J side, in agreement with the measurement of σ_{CF}, which necessites a critical value around $\ell_{cr}\hbar$ = 80 $\hbar$. We have already discussed that point.

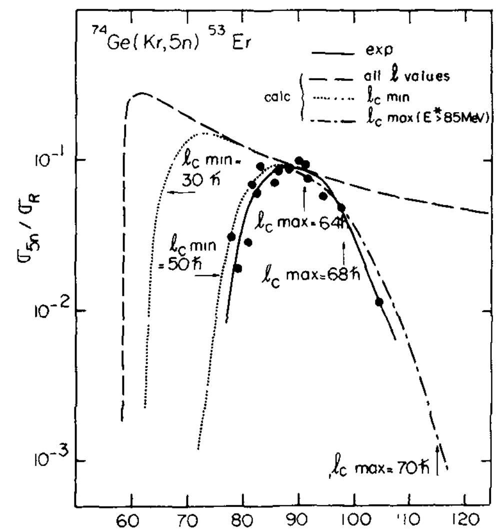

Fig. 23 - Attempt to fit krypton induced excitation function ^{74}Ge(Kr,5n)^{153}Er, by assuming critical angular momenta both in the lower side and in the upper side [76].

But for the Krypton case, the important shift implies necessarily a lower limit in the J population, evaluated around 50. (see figure 23).
The same cut-off should take place in order to describe the reactions (Kr,4n) and (Kr,6n) and a value slightly smaller was found [77] for the reaction $^{96}Zr(^{63}Cu,5n)^{154}Tm$ (J_{cr} around 35).

The same kind of energy shift was also observed recently [78] when comparing the excitation functions for polonium isotopes issued from the compound nucleus ^{200}Po after its formation either by $^{86}Kr+^{114}Cd$ or by $^{40}Ar+^{160}Dy$. In both cases, fission competition is very strong and high ℓ waves contribute entirely to fission, so the ℓ window for complete fusion becomes much narrower for Krypton projectiles than for Ar projectiles and cross sections for evaporation residues are smaller.

Such an hypothesis that low ℓ waves do not contribute to complete fusion for partners involving a large Z_1Z_2 product and therefore a high Coulomb potential, was made [29], in order to explain the lack of events in the low energy side of excitation functions as well as the very low complete fusion cross section in collisions between very heavy nuclei.
It is consistent with the idea that tangential friction occuring along a large loss of orbital angular momentum is necessary to allow a deeper penetration of highly charged projectiles into heavy targets, and subsequently to allow complete fusion. Although this is still an hypothesis, we arrive to the conclusion that complete fusion between complex nuclei can occur only with rather unexpected restrictive conditions, mainly a certain window in the ℓ wave population, as schematically shown on figure 24.

Perhaps we should mention that we have tried to find out other explanations. For example, if a high energy neutron was emitted in a preevaporation stage, there would be less excitation energy available, and the excitation function should be shifted towards higher values. But the preevaporation process occurs much more easily for light projectiles than for heavy ones, since, for the same excitation energy, incoming nucleons have a larger kinetic velocity. According to recent calculations by Blann [7], the probability for a preevaporated fast neutron is roughly 3-times larger for ^{12}C projectiles on ^{141}Pr than for ^{40}Ar on ^{109}Ag.

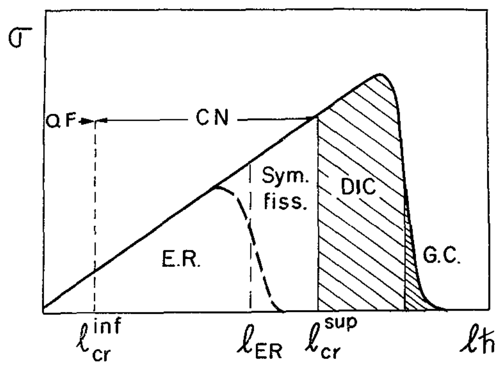

Fig. 24 - Schematic presentation of the different classes of reactions. Angular momentum population where ℓ cut-off are taken. For $\ell < \ell_{cr}^{inf}$, quasi-fission reactions occur. For $\ell_{cr}^{inf} < \ell < \ell_{cr}^{sup}$ complete fusion produces compound nuclei that may decay into evaporation residues or symmetric fission. For $\ell > \ell_{cr}^{sup}$ deep inelastic reactions (D.C.) are observed, and only for grazing collisions (G.C.) quasi-elastic scattering is occuring.

Another possibility could be a projection of a neutron during the collision time. Again such a "direct" process previous the compound nucleus formation occurs much more easily at 10 MeV pernucleon than at 2 or 3 MeV per nucleon. Furthermore, the phenomenon is well known for protons or ^3He projectiles, but the randomisation of the process, although it adds a long tail to excitation functions, does not displace the threshold on the low energy side.

The only suggestion left, outside of the low ℓ cut-off, could be that with massive projectiles, the head-on collision produces a collective vibration of the composite system and then a fraction of the energy available could be dissipated by gamma rays. No quantitative estimation of this possibility has been made.

It is perhaps not so surprising that complete fusion does not occur easily for massive projectiles and targets. In the early stage of deep inelastic collisions a strong damping is exerted and depending on the relative effects of nuclear + Coulomb potential, compound nucleus formation or re-separation of the two fragments after a large dissipation of energy may follow. In both cases, the reaction mechanism is complex, many nucleons are involved and the role played by collective degrees of freedom is important. The term "fusion" is correct since it means blending and melting under the effect of heat. "Complete fusion" corresponds to the long-lived compound nucleus formation where equilibration has occured for all degrees of freedom, whereas "incomplete fusion" corresponds to reactions where a "hot" metastable composite system is made and decays before full equilibration into two fragments. In that sense quasi-fission phenomena are typical of the second class, since equilibration has occured for energy, charge symmetry, but not for mass assymmetry. Furthermore for very heavy nuclei, all low and medium ℓ waves go into quasi-fission instead of contributing to complete fusion as they do for lighter systems.

5. Conclusion.

Complete Fusion and Energy Dissipation.

Then the end of my contribution might be taken as an introduction to Galin's report [12] on the damping process and its consequences in deep inelastic collisions.

Without entering too much into the subject, it is worthwhile to examine the reasons for the large hindrance factor which appears for complete fusion with heavy systems. It is related to two main causes: Conservative Coulomb forces, at the distance between centers d, expressed as ($Z_1Z_2/_{d}2$), and dissipative forces proportionnal to the average velocity during the overlap of the two nuclear matter densities. Such a velocity is difficult to evaluate but depends probably on ($\bar{E}$ - B), the energy above the barrier. Also, the dissipative forces should be proportionnal to the product of nuclear densities $\rho_1 \rho_2$,then to $1/d^2$. For DIC processes, J.Galin [12] introduces an interesting parameter,

$$\eta' = \frac{Z_1Z_2e^2}{\hbar \bar{v}_d} ,$$

similar to the Rutherford parameter η, except for $\bar{v}_d$ which is the average velocity when friction is exerted. It represents indeed the balance between Coulomb forces proportionnal to

$$\frac{Z_1Z_2e^2}{d^2}$$

and dissipative forces proportionnal to $\hbar\bar{v}_d/d^2$.

When η' is large, (Z_1Z_2 high and $\hbar\bar{v}_d$ small) there is such a strong tendancy against agglutination that no complete fusion occurs, even when the critical distance is reached.

When η' is smaller, then complete fusion becomes possible. For intermediate Z_1Z_2 values, $\hbar\bar{v}_d$ needs to be large enough, and therefore the incoming energy should be well above the barrier, for complete fusion occuring.

Now in a given system like (^{86}Kr + ^{74}Ge), $\bar{v}_d$ is certainly larger for intermediate impact parameters, than in head-on collisions, because the overlap of nuclear matter is not so big, and therefore η' might be sufficiently decreased so that complete fusion becomes available for a certain range of ℓ values. The same conclusion has been reached by Wilczynska and Wilczynski [79], using a repulsive core concept, and by Tsang [80], considering the effects of radial and tangential friction.

So far, we have oversimplified the picture by considering one dimensional potential energy curves. Nix and Moller [81] have calculated contour maps for the liquid drop potential energy as a function of two deformation axis.

Their main conclusion is the following :

a) As it has been stressed many time. by Swiatecki [82], binary fission and heavy-ion fusion take separate valleys. Therefore even if there is no descent inside the saddle point along the fission path, there still may be a ridge and a hollow inside that ridge along the fusion path.

b) The binary fission saddle point is always outside the contact point for light systems. It means that when the two nuclei are in contact and a neck developpes, the shape is less elongated than the shape at the saddle point, and therefore, the formation of a long-life compound system is possible. But for heavy systems, the saddle point shape is so close to the original spherical shape that it is inside the contact point. Therefore, even after a large energy relaxation, the system cannot do anything else than quasi-fission, as illustrated on figure 25. In Nix and Moller's prediction, the system ^{110}Pd + ^{110}Pd corresponds to the situation where the contact point and the saddle point are approximatively at the same distance, for ℓ = 0. This should exclude complete fusion for any symmetric system heavier than ^{110}Pd + ^{110}Pd. Although it has not been calculated, complete fusion should be possible for much heavier systems as far as they are asymmetric.

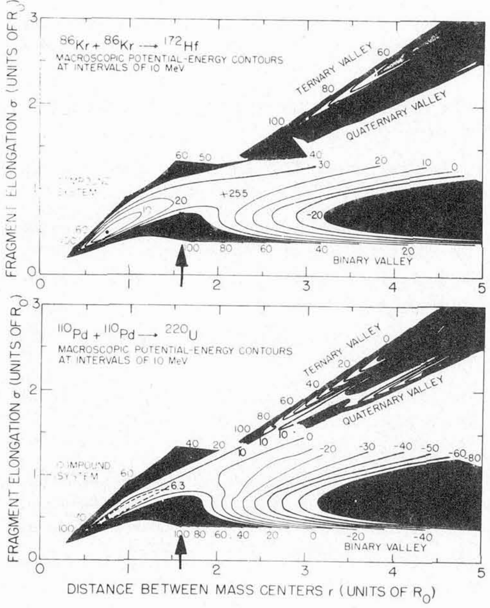

Fig. 25 - Macroscopic potential energy contour plots at $\ell = 0$ for the two systems [81] : a) $^{86}Kr+^{86}Kr \rightarrow ^{172}Hf$. The point of contact (●●) in H.I. reactions is well inside the binary saddle point. The dynamical path will proceed into the hollow on the left and a spherical shape will be attained, b) $^{110}Pd+^{110}Pd \rightarrow ^{220}U$. The binary saddle point now lies inside the point of first contact between the two H.I. Additional bombarding energy is required to form a compound nucleus.

In the sixfold way of interaction between heavy nuclei as defined by Swiatecki [83] one year ago, I have tried to treat the last category, compound system, although the frontier with the preceding category composite system is difficult to define very strictly.

Repeting Swiatecki's definition, nuclei that fuse but are not trapped form a composite system. (Fusion is defined in terms of a loss of identity of the pieces, associated with a filling in of the neck). The remainder are trapped in a potential energy hollow and form a compound system. I prefer myself to call it complete fusion in order to include in this category even those cases where full equilibrium has perhaps not been entirely reached for all the degrees of freedom, but however each piece have entirely lost its identity.

ORSAY, September 1976.

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